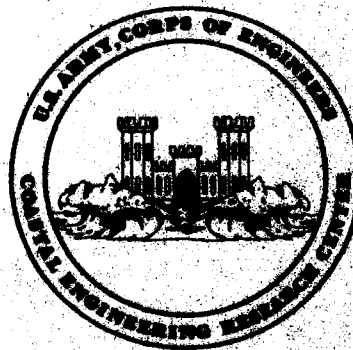END
DATE
FILMED
11.82
DTIC

Computer Algorithm to Calculate Longshore Energy Flux and Wave Direction from a Two Pressure Sensor Array

by

Todd L. Walton, Jr. and Robert G. Dean

TECHNICAL PAPER NO. 82-2

AUGUST 1982

DTIC
SELECTED
OCT 7
A

U.S. ARMY, CORPS OF ENGINEERS
COASTAL ENGINEERING
RESEARCH CENTER

Kingman Building
Fort Belvoir, Va. 22060

82 10 07 000

Reprint or republication of any of this material shall give appropriate credit to the U.S. Army Coastal Engineering Research Center.

Limited free distribution within the United States of single copies of this publication has been made by this Center. Additional copies are available from:

National Technical Information Service
ATTN: Operations Division
5285 Port Royal Road
Springfield, Virginia 22161

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TP 82-2	2. GOVT ACCESSION NO. AD-A119 980	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* COMPUTER ALGORITHM TO CALCULATE LONGSHORE ENERGY FLUX AND WAVE DIRECTION FROM A TWO PRESSURE SENSOR ARRAY		5. TYPE OF REPORT & PERIOD COVERED Technical Paper
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Todd L. Walton, Jr. Robert G. Dean		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of the Army Coastal Engineering Research Center (CEREN-EV) Kingman Building, Fort Belvoir, Virginia 22060		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS C31181
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army Coastal Engineering Research Center Kingman Building, Fort Belvoir, Virginia 22060		12. REPORT DATE August 1982
		13. NUMBER OF PAGES 33
14. MONITORING AGENCY NAME & ADDRESS*(if different from Controlling Office)*		15. SECURITY CLASS. *(of this report)* UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Computer program	Wave gage array
Longshore energy flux	Wave spectra
Sediment transport	

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A documented (FORTRAN IV) computer program is discussed as originally written for the CERC Longshore Sand Transport Research Program to analyze wave data collected at Channel Islands Harbor, California.

The program performs the basic analysis of two wave gage pressure records necessary to compute wave direction and wave energy at a given frequency and
(continued)

DD `FORM` `1 JAN 73` **1473** EDITION OF 1 NOV 65 IS OBSOLETE

computes the longshore energy flux used in sand transport for the entire energy spectrum of the wave record. This program uses linear wave theory for the wave transformation process and includes the assumption of straight and parallel bottom contours necessary for application of Snell's law of refraction.

The necessary steps in an analysis of wave data and sample outputs for some wave records from the Channel Islands wave gage pressure sensor pair are given. The program presently accepts data in the standard CERC magnetic-tape format where record lengths consist of 4,100 values.

PREFACE

This report provides coastal engineers with documentation necessary to compute the longshore energy flux used in sand transport rate calculation when random waves are present and synchronous data from two closely spaced pressure transducers exist. The documentation is based on a 3-year data collection effort and study of sand transport rates at Channel Islands Harbor, California. The computer program documented herein was used in wave data analysis for a two pressure sensor array installed in 30 feet of water at the site. The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Littoral Data Collection work unit, Shore Protection and Restoration Program, Coastal Engineering Area of Civil Works Research and Development.

This report was prepared by Dr. Todd L. Walton, Jr., Hydraulic Engineer, CERC, and Dr. Robert G. Dean, Department of Civil Engineering and College of Marine Studies, University of Delaware. Dr. Walton worked on the project under the general supervision of Dr. J.R. Weggel, Chief, Evaluation Branch, and Mr. N. Parker, Chief, Engineering Development Division.

Technical Director of CERC was Dr. Robert W. Whalin, P.E., upon publication of this report.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

3

CONTENTS

FIGURES

4

CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to
metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings,
use formula: C = (5/9) (F −32).

To obtain Kelvin (K) readings, use formula: K = (5/9) (F −32) + 273.15.

SYMBOLS AND DEFINITIONS

a_1, b_1	Fourier series coefficients
B	distance from bottom to pressure sensors
C_g	wave celerity
C_{12}	cospectrum value
d	total water depth
d_b	breaking wave depth
E	wave energy density
F	complex Fourier coefficient
f_n	discrete frequency value
GB,GBP	ratio of rms breaking wave height to breaking wave depth
g	acceleration of gravity
H_b	breaking wave height
i,j	counting indexes
K_z	dynamic pressure response factor
k	wave number
L	wavelength
ℓ	sensor spacing
m	index to account for gage number
N	total number of discrete data points
n	frequency number, argument of Fourier series coefficients
$P_{\ell s}$	longshore energy flux at the surf line
P_1	pressure time-series values
p	dynamic pressure
Q_{12}	quad-spectrum value
R	ratio of unwindowed energy density to windowed energy density
S_{12}	complex cross-spectrum value
T	length of time series record
T_{HF}	high frequency cutoff period
w	weighting coefficient
z	water surface elevation
β	gage orientation angle
γ	specific weight of seawater
θ	wave direction
Δd	average mean depth of water overlaying pressure sensors
Δf	frequency step
Δt	time step
ω	angular wave frequency

6

COMPUTER ALGORITHM TO CALCULATE LONGSHORE ENERGY FLUX AND WAVE DIRECTION FROM A TWO PRESSURE SENSOR ARRAY

by
Todd L. Walton, Jr. and Robert G. Dean

I. INTRODUCTION

The documented (FORTRAN IV programing language) computer program discussed in this report was originally written as part of the Coastal Engineering Research Center's (CERC) Longshore Sand Transport Research Program and was used in analysis of wave data collected at Channel Islands Harbor in conjunction with a study of sand transport at Channel Islands Harbor as discussed in Bruno, et al. (1981).

The program performs the basic analysis of two wave gage pressure records necessary to compute wave direction and wave energy at a given frequency and computes the longshore energy flux used in sand transport for the entire energy spectrum of the wave record. This program uses linear wave theory for the wave transformation process and includes the assumption of straight and parallel bottom contours necessary for application of Snell's law of refraction.

Necessary steps in the analysis of the wave data are presented in Sections II and III of this report. Subroutines are discussed and sample outputs for some wave records from the Channel Islands wave gage pressure sensor pair are given.

The program presently accepts data in the standard CERC magnetic-tape format where record lengths consist of 4,100 values. The first four values are the gage number and the date-time group, and the remaining 4,096 values are the pressures recorded in thousandths of a foot (head) of water at 0.25-second intervals. Should other input data be available, the program could easily be modified to accept the data by simple changes in the main program and in subroutines BUF and SWITCH.

Sample outputs have been presented for real wave data; some wave directional information cannot be obtained for all frequencies because the spectral information at some frequencies is ill-conditioned. The percent of energy for which this problem occurs is a small part of the energy (usually <3 percent) of the entire spectrum and is insignificant in energy-flux computations. Reasons for this feature are discussed later.

II. METHODOLOGY

Calculating the longshore energy flux at breaking required the following steps:

(1) Calculation of the frequency-by-frequency wave direction and energy at the location of the wave gages;

(2) determination of the breaking wave depth;

(3) transformation of the wave spectrum to the "breaker" line, including shoaling and refraction effects; and

(4) computation of "$P_{\ell s}$," the longshore energy flux at the surfline.

Each of the steps is described below.

1. Calculation of Wave Direction and Energy Spectrum at Wave Gages.

As noted previously, each of the input time-series pressure records con-
sists of 4,096 data points with a time increment of 0.25 second. To reduce
computational costs, modified time series are formed for analysis by averaging
four adjacent data points. These new time series contain 1,024 data points
spaced at 1.0-second intervals. This increases the aliasing period from 0.5
to 2.0 seconds; however, this is justified as the pressure response factor
for a water depth of 6 meters and a wave period of 2 seconds is approximately
0.005.

The time series are analyzed using a standard fast Fourier transform (FFT)
program to determine the coefficients. For example, for pressure time series
from gage 1

$$P_1(j) = \sum_{n=0}^{N-1} [a_1(n) - ib_1(n)] \exp\left(\frac{i2\pi nj}{N}\right) \tag{1}$$

in which $i = \sqrt{-1}$ and N is the total number of data points, $T/\Delta t = 1,024$,
where T is the time series record length of 1,024 seconds, Δt the time
increment of 1 second between samples, and j a discrete time t_j where $t_j =$
discrete time value $= j\Delta t$. The FFT coefficients are defined in terms of the
pressure time series as

$$a_1(n) - ib_1(n) = \frac{1}{N} \sum_{j=0}^{N-1} P_1(j) \exp\left(-i\frac{2\pi nj}{N}\right) \tag{2}$$

where the argument "n" of the Fourier coefficients $a(n)$ and $b(n)$ speci-
fies the quantity to be a discrete function of wave frequency, f_n, where
f_n, a discrete frequency value, is $n\Delta f$ (where $\Delta f = 1/T$) and the $a_1(0)$ term
represents the mean value of the time-series pressure record for wave gage 1.
Similar relationships exist for wave gage 2. In calculating the FFT coeffi-
cients, there are several options that may be employed in an attempt to reduce
spectral leakage which arises due to representing an aperiodic time series by
a periodic series. A large number of possible data windows (weighting func-
tions for data) have been developed to reduce the adverse effects of spectral
leakage (Harris, 1974). These can be expressed in the form of a weighting
function $w(j)$, such that the modified time series $p'(j)$ is of the form

$$p'(j) = w(j) \ p(j)$$

in which $p(j)$ is the digitized measured pressure value at time $t_j = j\Delta t$, and
$w(j)$ a weighting function. A characteristic of these weighting functions is
that they are equal to unity at the midpoint of the time series and decrease
to a lesser value near the two ends. In the present program, a "cosine bell"
weighting function is used; however, through comparisons of P_{es} with and
without this function, it was established that the effect of the weighting
function was minimal (<5 percent). The cosine bell weighting function is
expressed by

$$w(j) = \frac{1}{2}\left(1.0 - \cos\frac{2\pi j}{N}\right) \tag{3}$$

8

It is clear that the application of a weighting function will reduce the total energy in the record. This effect is partly compensated for by the following equation:

$$p''(j) = \sqrt{\frac{\langle p^2 \rangle}{\langle p'^2 \rangle}} \, p'(j) \qquad (4)$$

thereby ensuring the same total energy in the altered and original time series, where $\langle p^2 \rangle$ is the mean square value of the original time series and $\langle p'^2 \rangle$ the mean square value of the weighted time series. It is the altered time series $p''(j)$ that is subjected to FFT analysis. The primes will be dropped hereafter for convenience. The average mean depth of water overlying the pressure sensors, Δd, is obtained by averaging the m time series to obtain $a_m(0)$. For two separate time series records, m = 1, 2 (wave gages 1 and 2),

$$\Delta d = 0.5 \, [a_1(0) + a_2(0)] \qquad (5)$$

The total water depth, d, is the sum of Δd and the distance, B, of the pressure sensors above the bottom (in later examples B $\simeq$ 0.76 meter).

Each FFT pressure coefficient is transformed to a water surface displacement coefficient by the following linear wave theory relationship discussed in the Shore Protection Manual (SPM) (see Ch. 2, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977):

water surface coefficients   dynamic pressure coefficients

$$[a_m(n), \, b_m(n)]_\eta = \frac{1}{\gamma K_z(n)} \, [a_m(n), \, b_m(n)]_p \qquad (6)$$

in which the subscripts η and p denote water surface and dynamic pressure coefficients, respectively. The factor

$$K_z(n) = \frac{\cosh k(n) \, B}{\cosh k(n) \, d} \qquad (7)$$

where γ is the specific weight of fluid (seawater) and is included when pressure coefficients are in normal units of pressure (i.e., N/M^2 or equivalent). In equation (7), B represents the distance of the pressure sensors above the bottom and k(n) is the wave number associated with the angular frequency, $\omega(n) = (2\pi n \Delta f)$, as obtained from the linear wave theory dispersion relationship

$$\omega(n)^2 = gk(n) \tanh k(n) \, d \qquad (8)$$

One of the disadvantages of measuring waves with near-bottom pressure sensors is evident by examining equations (6) and (7). For the higher frequencies (shorter wave periods) $K_z(n)$ is very small which means that the higher frequency waves result in very small pressure fluctuations near the sea floor. Thus, to avoid contaminating the calculated water surface displacements, it is

9

usually necessary to apply a high frequency cutoff, above which the pressure contributions are discarded. The proper selection of this high frequency cutoff depends on the signal to noise characteristics of the pressure sensor and the signal conditioning system. In the present program, the high frequency cutoff was established at a wave period of 3.0 seconds. Wave gage analyses by Thompson (1980) have shown that a 3.0-second high frequency spectral cutoff value provides reasonable estimates of total wave energy at west coast (U.S.) locations.

Denoting hereafter the FFT coefficients for the water surface as $a(n)$ and $b(n)$, it is noted that the coefficients have the following properties:

$$\langle \eta^2 \rangle = \sum_{n=1}^{N-1} [a^2(n) + b^2(n)] \tag{9}$$

and

$$a\left(\frac{N}{2} + n\right) = a\left(\frac{N}{2} - n\right) \tag{10}$$

$$b\left(\frac{N}{2} + n\right) = -b\left(\frac{N}{2} - n\right) \tag{11}$$

and thus

$$\langle \eta^2 \rangle = 2 \sum_{n=1}^{N/2} [a^2(n) + b^2(n)] \tag{12}$$

Thus, the total (kinetic and potential) energy $E(n)$ associated with a particular wave frequency component, n, is

$$E(n) = 2\gamma[a^2(n) + b^2(n)] \tag{13}$$

Now consider two wave or pressure sensors located at (x_1, y_1) and (x_2, y_2) (see Fig. 1). The results will be developed considering discrete frequencies.
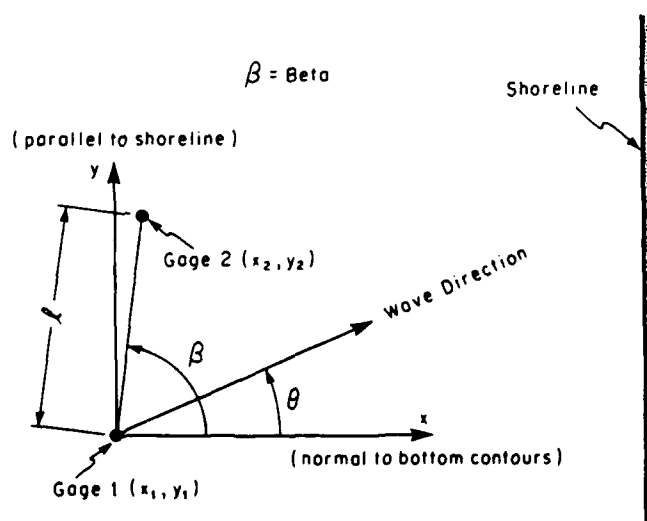

Figure 1. Definition sketch for two sensor array.

The water surface displacement consistent with the assumption of one direction per frequency is

$$\eta(x, y, j) = \sum_{n=0}^{N-1} F(n) \exp\{i[n\omega_1 t - k_x(n) x - k_y(n) y]\}$$

$$= \sum_{n=0}^{N-1} [a(n) - ib(n)] \exp\left(\frac{i2\pi nj}{N}\right)$$

(14)

where ω_1 is the primary analysis frequency ($= 2\pi/\text{record length} = 2\pi/T = 2\pi\Delta f$), and $\Theta(n)$ the direction of wave propagation at frequency $\omega(n) = n\omega_1$. The wave number components, $k_x(n)$ and $k_y(n)$, are expressed in terms of the wave number, $k(n)$, and wave direction, $\Theta(n)$, as

$$k_x(n) = k(n) \cos \Theta(n) \qquad (15)$$

$$k_y(n) = k(n) \sin \Theta(n) \qquad (16)$$

The cross spectrum, $S_{12}(n)$, of the two measured water surface displacements (or dynamic pressures) is given by

$$S_{12}(n) = |F(n)|^2 \{\exp - i [k(n) \cos \Theta(n)(x_2 - x_1)$$

$$+ k(n) \sin \Theta(n)(y_2 - y_1)]\}$$

(17)

Denoting the separation distance and angle as ℓ and β, respectively, the cross spectrum can be expressed as (see Fig. 1)

$$S_{12}(n) = |F(n)|^2 \{\cos [k(n) \ell\cos (\Theta(n) - \beta)] - i \sin [k(n) \ell\cos (\Theta(n) - \beta)]\}$$

$$= \text{cospectrum } (n) - i \text{ quad-spectrum } (n) \qquad (18)$$

$$= C_{12}(n) - iQ_{12}(n)$$

Thus, from equation (18), the wave direction $\Theta(n)$ associated with each wave frequency can be expressed as

$$\Theta(n) = \beta \mp \cos^{-1}\left\{\frac{1}{k(n)\,\ell} \tan^{-1}\left[\frac{Q_{12}(n)}{C_{12}(n)}\right]\right\} \qquad (19)$$

The above relationship has two roots, one of which must be selected based on physical considerations of the most likely direction of wave propagation. In the present case, assuming no wave reflection from the beach, the ambiguity in wave direction is ruled out; for wave sensors nearly parallel to the beach, the minus sign in equation (19) is appropriate.

11

There are two conditions for which it was not possible to calculate the wave directions $\Theta(n)$. These include poorly conditioned wave data, presumably due to spectral leakage, and spatial aliasing due to large separation distance between the two gages. If the data are poorly conditioned for determining wave direction, the absolute value of the quantity within the brackets {-} in equation (19) may exceed unity, a physically impossible condition since the extreme values of the cosine function are ±1. This tends to occur for the extremely long waves for which the energy is small and the value of $k(n)$ is also small, the latter tending to result in large values of the bracketed quantity. The percentage of energy for which this condition occurred in the analysis of one year's wave data collected at Channel Islands Harbor was relatively small, averaging 2 to 3 percent with a maximum of approximately 10 percent. The second condition is related to spatial aliasing and requires that one-half the wavelength be equal to or greater than the projection of the wave gage separation distance in the direction of wave propagation. Referring to Figure 1,

$$L > 2\ell \ \{\cos[\Theta(n) - \beta]\}_{max} \tag{20}$$

which indicates that for the least adverse effects of spatial aliasing, the gages should be on an alinement parallel to the dominant orientation of the wave crests. As will be discussed later, in calculating $P_{\ell s}$ an attempt was made to account for this effect of aliasing by augmenting the calculated values, illustrated as follows by

$$(P_{\ell s})_{cm} = (P_{\ell s})_c \ \frac{E_{TOT}}{E} \tag{21}$$

in which the subscripts c and cm indicate calculated and calculated modified, respectively. E_{TOT} and E represent the total wave energy values and the wave energy not affected by spatial aliasing or poorly conditioned wave data, respectively. The total wave energy is that energy in the wave spectrum below the high frequency spectral cutoff value.

2. Transformation of Wave Spectrum to Breaker Line.

At this stage, the wave energy and wave direction in the vicinity of the gages are determined. These values are then transformed to the breaker line accounting for wave refraction and shoaling.

To determine the wave breaking depth, the onshore-directed energy flux is calculated in accordance with the expression (based on Snell's law of refraction) and equated to an equivalent expressed in terms of wave characteristics at breaking.

$$\text{Onshore energy flux} = \sum_{n-1}^{N/2} \gamma 2 \ [a(n)^2 + b(n)^2] \ C_g(n) \ \cos \Theta(n)$$

$$= \frac{\gamma E_b^2}{8} \ C_{gb} \ \cos \Theta_b \tag{22}$$

12

Assuming that the breaking wave angle, Θ_b, is small, that the waves will break under shallow-water conditions, and that the ratio of breaking wave height to depth is a constant, the breaking wave height, H_b, is then given by

$$H_b = \left\{ \sum_{n=1}^{N/2} 16 \left[a(n)^2 + b(n)^2 \right] C_g(n) \cos \Theta(n) \right\}^{0.4} \left(\frac{GB}{g} \right)^{0.2} \tag{23}$$

where GB is the ratio of root-mean-square (rms) breaking wave height to breaking depth, $GB = H_b/d_b$ (here assumed GB = 0.78). With the breaking depth known, each wave component is transformed to shore accounting for both wave refraction and shoaling based on linear wave theory.

Wave refraction is in accordance with Snell's law and the assumption that straight and parallel contours existed between the gage and breaking locations

$$\Theta_b(n) = \sin^{-1} \left[\frac{C_b}{C_r(n)} \right] \sin \Theta_r(n) \tag{24}$$

where C is linear wave celerity (see the SPM, Ch. 2) in which the r subscripts denote the "reference (gage)" location.

With the wave energy and direction now known at the breaker line, the value of the longshore energy flux, $(P_{\ell s})_{cm}$, is readily determined

$$(P_{\ell s})_{cm} = R(P_{\ell s})_c$$

$$= R \left\{ 2\gamma \sum_{n=1}^{N/2} \left[a^2(n) + b^2(n) \right]_b \left[C_g(n) \right]_b \left[\cos \Theta(n) \sin \Theta(n) \right]_b \right\} \tag{25}$$

in which the factor R is given by the ratio

$$R = \frac{E_{TOT}}{E}$$

as defined in and discussed in relation to equation (21).

III. MAIN PROGRAM DOCUMENTATION

The detailed programing steps in analysis for the longshore energy flux, $(P_{\ell s})_{cm}$, (which in this program is calculated in terms of rms wave height) are presented in this section. Program steps are numbered to correspond to areas in the program listing where computations are carried out. A program listing with corresponding numbered steps follows the program documentation. Note that preceding text has used the indexes j and n for time and frequency, respectively, while the program which follows uses the index I for both time and frequency. A listing of the main program is presented in Figure 2. Program steps are as follows and refer to numbered parts of main program listing:

13

```
1          PRUGKAM SPECT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE0)
        C  CUMPUTOR ALGORITHM TU CALCULATE LONGSHORE ENERGY FLUX FACTOR AND WAVE
        C     DIRECTION FOR TWO PRESSURE SENSOR ARRAY
        C  MAIN PROGRAM
5       C  PRUGRAM IS PRESENTLY SET UP TU TAKE A TIME SERIES OF 1024 POINTS IN MAIN
           DIMENSION C(512)
           DIMENSION F1R(1024),F1I(1024),F2R(1024),F2I(1024)
           DIMENSION SIGMA(512),FMODSQ(512),THETA(512)
           DIMENSION C12(512),Q12(512)
10         DIMENSION W(1024)
           DIMENSION CG(512),SINTHS(512)
           REAL MEAN1,MEAN2
           LOGICAL END
           DATA END/.FALSE./
15      101 FORMAT(10(2X,F8.2))
        C
        C
        C   DEFINITIONS=FIXED VARIABLES
        C   K=EXPONENTIAL POWER DEFINING NUMBER OF TIME SERIES POINTS=(2**K)
20      C   S=SPACING BETWEEN WAVE GAGES (FEET)
        C   DELTT=TIME STEP BETWEEN POINTS IN AVERAGED TIME SERIES (SECONDS)
        C   BETA =ANGLE DIFFERENCE BETWEEN WAVE GAGE ALIGNMENT AND SHORELINE(RADIANS)
        C   SLOPE=SLOPE OF REACH AT PUINT OF WAVE BREAKING
        C   GAMMA=SPECIFIC WEIGHT OF FLUID (LBS/FT**3)
25      C   B=DISTANCE OF PRESSURE SENSORS ABOVE BOTTOM (FEET)
        C   G=ACCELERATION OF GRAVITY (FEET/SEC**2)
        C   GB=RATIO BREAKING WAVE HEIGHT/DEPTH FOR LINEAR THEORY COMPUTATION
        C      OF WAVE HEIGHT
        C   GBP=RATIO BREAKING WAVE HEIGHT/DEPTH FOR LINEAR THEORY COMPUTATION OF
30      C      WATER DEPTH GIVEN BREAKING WAVE HEIGHT
        C
        C
        C   DEFINITIONS=FLOATING VARIABLES
        C   AVG1=AVERAGE OF TIME SERIES1
35      C   AVG2=AVERAGE OF TIME SERIES 2
        C   C(I)=WAVE CELERITY
        C   C12(I)=COSPECTRA OF SERIES1-2
        C   CB=BREAKING WAVE CELERITY
        C   CG(I)=GROUP WAVE CELERITY
40      C   CNTL(I)=4096 POINT TIME SERIES BEFORE AVERAGING
        C   DEPTH=DEPTH OF WATER AT GAGE SITE FROM AVERAGES OF GAGES 1 AND 2
        C   F1I(I)=UNDEFINED/COMPLEX IMAGINARY PORTION OF TRANSFORM
        C   F1R(I)=TIME SERIES DATA GAGE1/COMPLEX REAL PORTION OF TRANSFORM
        C   F2I(I)=UNDEFINED/COMPLEX IMAGINARY PURTION OF TRANSFORM
45      C   F2R(I)=TIME SERIES DATA GAGE2/COMPLEX REAL PORTION OF TRANSFURM
        C   FMODSQ(I)=TIME SERIES AMPLITUDE MODULUS SQUARED
        C   MB=BREAKING WAVE HEIGHT
        C   IA(I)=5000 POINT DATA GROUP AND TIME SERIES RECORD
        C   PLNEG=NEGATIVE CONTRIBUTION TO LONGSHORE ENERGY FLUX FACTOR
50      C   PLNET=NET LONGSHORE ENERGY FLUX FACTOR
        C   PLPOS=POSITIVE CONTRIBUTION TO LONGSHORE ENERGY FLUX FACTOR
        C   Q12(I)=QUADSPECTRA OF SERIES 1-2
        C   R=SCALING FACTOR FOR SCALING UP ENERGY OF NONUSABLE
        C      PORTIONS OF DIRECTIONAL SPECTRA
55      C   RATIO1=RATIO OF ENERGY/WINDOWED ENERGY FUR GAGE 1
        C   RATIO2=RATIO OF ENERGY/WINDOWED ENERGY FUR GAGE 2
        C   REAL(I)=1024 POINT TIME SERIES AFTER AVERAGING
        C   RN=RATIO OF GROUP WAVE CELERITY TO WAVE CELERITY
        C   RSHFRQ=PERCENT OF ENERGY BEYOND SPACIAL ALIASING FREQUENCY
60      C   RSLFRQ=PERCENT OF ENERGY BELOW LOW FREQUENCY CUTOFF
        C   RSUDU=PERCENT OF INCOHERENT ENERGY
        C   SHFREQ=SUM OF ENERGY WITH FREQUENCIES ABOVE SPACIAL
        C      ALIASING FREQUENCY CUTOFF
        C   SHG2=SUMMATION OF ONSHORE ENERGY FLUX
65      C   SIGMA(I)=RADIAL FREQUENCY
        C   SLFREQ=SUM OF ENERGY WITH FREQUENCIES BELOW LOW FREQUENCY CUTOFF
        C   SUDU=SUM OF ENERGY WITH FREQUENCIES HAVING INCOHERENT WAVE DIRECTION
        C   SUMEN=SUM OF ENERGY
        C   SUM1=SUM OF SQUARES OF TIME SERIES 1 WITHOUT AVERAGE
70      C   SUM2=SUM OF SQUARES OF TIME SERIES 2 WITHOUT AVERAGE
        C   SUMF1=SUM OF SQUARES OF TIME SERIES 1 WITH AVERAGE
        C   SUMF2=SUM OF SQUARES OF TIME SERIES 2 WITH AVERAGE
        C   T=WAVE PERIOD
        C   THETA(I)=WAVE DIRECTION IN RADIANS
```

Figure 2.   Listing of main program.

14

```
                C     THETAB=BREAKING WAVE ANGLE
                C     WSUM1=SUM OF SQUARES OF DATA WINDOW MODIFIED TIME SERIES 1
                C     WSUM2=SUM OF SQUARES OF DATA WINDOW MODIFIED TIME SERIES 2
  75                  PI=3.14159265
                      TWOPI=2.0*PI
  80                  K=10
                      N=2.**K
                      S=80.0
                      DELTT=1.00
                      BETA=1.5708
  85                  SLOPE=0.05
                      GAMMA=64.0
                      B=2.5
                      MON=1
                      NU2=N/2
  90                  GB=0.78
                      G=32.2
                      GBP=0.78

                C
                C     HIGH FREQ CUTOFF=3.0 SEC
  95            C     SPACIAL ALIASING CUTOFF=3.4 SEC
                C     NLOW=LOW FREQUENCY CUTOFF NUMBER
                C     NYFR=HIGH FREQUENCY CUTOFF NUMBER
                C     NSALFR=SPACIAL ALIASING FREQUENCY CUTOFF NUMBER
                C     FREQUENCY CUTOFF NUMBER=TIME SERIES LENGTH/CUTOFF PERIOD
 100            C
                      NLOW=50
                      NYFR=342
                      NSALFR=301
                      NSM1=NSALFR-1
 105            110 CONTINUE

                C
                C     INITIALIZING VALUES
                      SLFREQ=0.0
                      SUDD=0.0
 110                  SHFREQ=0.0
                      SUMEN=0.0
                      SUM1=0.
                      SUM2=0.
                      SUMF1=0.

 115                  SUMF2=0.
                      WSUM1=0.0
                      WSUM2=0.0
                      AVG1=0.0
                      AVG2=0.0

 120                  PLPOS=0.0
                      PLNEG=0.0
                      PLNEI=0.0
                      SMG2=0.0
                      DO 29 I=1,N
 125                  F1I(I)=0.0
                      F2I(I)=0.0
                  29 CONTINUE
                      DO 30 I=1,NU2
                      FMODSQ(I)=0.0
 130             30 CONTINUE

                C
                C     THIS PORTION OF PROGRAM READS IN WAVE PRESSURE VALUES INTO F1R,F2R ARRAYS
                C     AND ASSURES MATCHING DATE GROUPS FOR DIRECTIONAL WAVE ANALYSIS OF TWO
                C     GAGES.
 135                  CALL BUF(MGAGE1,MONTH1,MDAY1,MTIME1,F1R  ,IDATE1,END)
                      IF(END) GO TO 1
                      CALL BUF(MGAGE2,MONTH2,MDAY2,MTIME2,F2R  ,IDATE2,END)
                      IF(END) GO TO 1
                      IF(IDATE1.EQ.IDATE2) GO TO 120
```

Figure 2. Listing of main program.--Continued

15

```
140           BACKSPACE 9
              GO TO 110
          120 CONTINUE
              IF(MGAGE1.NE.311) CALL SWITCH(MGAGE1,MGAGE2,F1R  ,F2R  )
              WRITE(6,426)
145       426 FORMAT(//,[ GAUGE NO.[,6X,[MONTH[,7X,[DAY[,8X,[TIME[)
              WRITE(6,11)MGAGE1,MONTH1,MDAY1,MTIME1
              WRITE(6,11)MGAGE2,MONTH2,MDAY2,MTIME2
           11 FORMAT(I7,3(5X,I7))


          C
          C
150       C     THIS PORTION OF PROGRAM CALCULATES WATER DEPTH AT WAVE GAGES AS WELL AS
          C       AVERAGES AND SUM OF SQUARES OF TIME SERIES
              DO 42 I=1,N
              AVG1=AVG1+F1R(I)
           42 AVG2=AVG2+F2R(I)
155           AVG1=AVG1/FLOAT(N)
              AVG2=AVG2/FLOAT(N)
              DEPTH=(AVG1+AVG2)/2.+B
              CALL HFC(DEPTH,B,DELTT,N,NSALFR)
              DO 41 I=1,N
160           F1R(I)=F1R(I)-AVG1
              F2R(I)=F2R(I)-AVG2
              SUM1=SUM1+F1R(I)**2.
              SUM2=SUM2+F2R(I)**2.
           41 CONTINUE
165           SUM1=SUM1/FLOAT(N)
              SUM2=SUM2/FLOAT(N)


          C
          C     THIS PORTION OF PROGRAM APPLIES DATA WINDOW TO TIME SERIES--DATA WINDOW
          C       VALUES ARE REPRESENTED BY W(I)
170           DO 89 I=1,N
              W(I)=0.5*(1.0-COS(TWOPI*FLOAT(I)/FLOAT(N)))

              F1R(I)=(F1R(I)       )*W(I)
              F2R(I)=(F2R(I)       )*W(I)
           89 CONTINUE


175       C
          C     THIS PORTION OF PROGRAM COMPUTES SUM OF SQUARES OF DATA WINDOW MODIFIED
          C       TIME SERIES AS WELL AS RATIO OF PRE WINDOWED ENERGY TO WINDOWED ENERGY
              DO 43 I=1,N
              WSUM1=WSUM1+F1R(I)**2.
180           WSUM2=WSUM2+F2R(I)**2.
           43 CONTINUE
              WSUM1=WSUM1/FLOAT(N)
              WSUM2=WSUM2/FLOAT(N)
              RATIO1=SUM1/WSUM1
185           RATIO2=SUM2/WSUM2
              CALL FFT(F1R,F1I,K,0)
              CALL FFT(F2R,F2I,K,0)
              MEAN1=F1R(1)
              MEAN2=F2R(1)


190       C
          C     THIS PORTION OF PROGRAM CALCULATES CO AND QUAD SPECTRA VALUES, AS WELL AS
          C       WAVE ANGLE TO SHORELINE AND ENERGY CONTRIBUTIONS OF EACH FREQUENCY,
          C       BREAKING WAVE HEIGHT AND BREAKING WAVE CELERITY ARE ALSO
          C       CALCULATED IN THIS SECTION
195           I=1
              DO 97 J=2,N
              F1R(I)=F1R(J)
              F1I(I)=F1I(J)
              F2R(I)=F2R(J)
200           F2I(I)=F2I(J)
              I=I+1
           97 CONTINUE
              DO 96 I=1,M
              SUMF1=SUMF1+F1R(I)**2.+F1I(I)**2.
```

Figure 2. Listing of main program.--Continued

16

```
205           SUMF2=SUMF2+F2R(I)**2.+F2I(I)**2.
           96 CUNTINUE
              SUMF1=SUMF1+MEAN1**2.
              SUMF2=SUMF2+MEAN2**2.
              WRITE(6,2A9)
210       289 FURMAT(//,7X,(I(,10X,(SIGMA(I)(,11X,(FMODSQ(I)()
              DU 99 I=1,ND2
              C12(I)=F1R(I)*F2R(I)+F1I(I)*F2I(I)
              U12(I)=F1R(I)*F2I(I)-F2R(I)*F1I(I)
              SIGMA(I)=FLOAT(I)*THUPI/(FLOAT(N)*DELTT)
215           T=THUPI/SIGMA(I)
              CALL WVLEN(DEPTH,T,XKH)
              XK=XKH/DEPTH
              IF(C12(I).LE.0.000000001) GO TU 95
              PD=(1./((XK*SJ)*ATAN(U12(I)/C12(I)))
220           IF(ABS(PD).GT.1.0) GO TU 93
              THETA(I)=-ACOS(PD)+BETA
              GO TU 92
           95 THETA(I)=0.0
              GU TU 92
225        93 THETA(I)=0.00001
              FMODSU(I)=F1R(I)**2.+F1I(I)**2.
              XKH=XKH*B/DEPTH
              XKP=CUSH(XKH)/COSH(XKH)
              FMODSU(I)=FMODSQ(I)/(XKP**2.)
230           FMODSU(I)=FMODSQ(I)*RATIU1
              SUDD=SUDD+FMODSQ(I)
              WRITE(6,105)I,SIGMA(I),FMODSU(I)
          105 FURMAT(3X,I5,5X,F12.6,7X,F12.6)
           92 CONTINUE
235           FMODSU(I)=F1R(I)**2.+F1I(I)**2.
              XKH=XKH*B/DEPTH
              XKP=CUSH(XKH)/COSH(XKH)
              FMODSU(I)=FMODSQ(I)/(XKP**2.)
              FMODSU(I)=FMODSQ(I)*RATIO1
240           RN=0.5*(1.+2.*XKH/SINH(2.*XKH))
              CG(I)=SIGMA(I)*DEPTH*RN/XKH
              C(I)=CG(I)/RN
              MG2=(CG(I)*2.0*FMODSU(I)*COS(THETA(I)))
       C      SMG2=ONSHORE ENERGY FLUX
245           SMG2=SMG2+MG2
              SUMEN=SUMEN+FMODSU(I)
              IF(I.GE.NBALFR) GO TU 79
              GU TU 78
           79 SHFREQ=SHFREQ+FMODSQ(I)
250        78 CUNTINUE
              IF(I.LE.NLOW) GO TO 77
              GU TU 76
           77 SLFREQ=SLFREQ+FMODSQ(I)
           76 CUNTINUE
255           IF(I.GE.NYFR) GO TO 999
           99 CONTINUE
          999 CONTINUE
              SMG2=SMG2*2.
              WRITE(6,351)
260       351 FURMAT(//,6X,(I(,18X,(SIGMA(I)(,11X,(PCT(,16X,(THETA(I)()
              DU 48 I=1,ND2
              IF(I.GE.NBALFR) GO TO 48
              PCT=FMODSQ(I)/SUMEN
              IF(PCT.GE.0.025) GO TO 49
265           GO TU 48
           49 WRITE(6,50)I,SIGMA(I),PCT,THETA(I)
           50 FURMAT(3X,I5,3(3X,F16.8))
           48 CUNTINUE
           44 CUNTINUE
```

Figure 2. Listing of main program.--Continued

17

```
270         HB=(8.**.4)*(SHG2**.4)*(GB/G)**.2
            CB=(G*HB/GBP)**.5
         57 CONTINUE


        C
        C      THIS PORTION OF PROGRAM MODIFIES WAVE GAGE ANGLES TO BREAKING WAVE ANGLES
275     C         AND COMPUTES LONGSHORE ENERGY FLUX FACTORS
            DO 91 I=1,ND2
            IF(I.GE.NSALFR) GO TO 998
            SINTHB(I)=SIN(THETA(I))*CB/C(I)
            THETAB=ASIN(SINTHB(I))
280         XKRS=((1.-SIN(THETA(I))**2.)/(1.-SINTHB(I)**2.))**.5
            XKSS=CG(I)/CB
            FMODSQ(I)=FMODSQ(I)*XKRS*XKSS
            IF(THETA(I).LE.0.0) GO TO 87
            PLPOS=PLPOS+GAMMA*SIN(2.*THETAB   )*CB*FMODSQ(I)
285         GO TO 88
         87 PLNEG=PLNEG+GAMMA*SIN(2.*THETAB   )*CB*FMODSQ(I)
         88 CONTINUE
            PL=GAMMA*CB*SIN(2.*THETAB   )*FMODSQ(I)
            PLNET=PLNET+PL
290         IF(I.GE.NYFR) GO TO 998
         91 CONTINUE
        998 CONTINUE
            RSODD=SODD/SUMEN
            RSHFRQ=SHFREQ/SUMEN
295         RSLFRQ=SLFREQ/SUMEN
            RTOT=RSODD+RSHFRQ
            R=1./(1.-RTOT)
            PLPOS=PLPOS*R
            PLNEG=PLNEG*R
300         PLNET=PLNET*R
            WRITE(6,201)NSALFR
        201 FORMAT(//,( NSALFR=(,24X,I6)
            WRITE(6,200)DEPTH
        200 FORMAT(( DEPTH OF WATER AT GAUGE SITE=(,F10.1)
305         WRITE(6,100)AVG1,AVG2
        100 FORMAT(( AVG1=(,F11.3,9X,(AVG2=(,F11.3)
            WRITE(6,170)SUM1,SUM2
        170 FORMAT(( SUM1=(,F11.3,9X,(SUM2=(,F11.3)
            WRITE(6,111)WSUM1,WSUM2
310     111 FORMAT(( WSUM1=(,F10.3,9X,(WSUM2=(,F10.3)
            WRITE(6,112)RATIO1,RATIO2
        112 FORMAT(( RATIO1=(,F9.3,9X,(RATIO2=(,F9.3)
            WRITE(6,39)SUMEN
         39 FORMAT(( SUMEN=(,2X,F13.8)
315         WRITE(6,104)HB
        104 FORMAT(( BREAKING WAVE HEIGHT HB=(,6X,F10.2)
            WRITE(6,108)CB
        108 FORMAT(( BREAKING WAVE CELERITY CB=(,4X,F10.2)
            WRITE(6,106) RSODD,RSHFRQ,RSLFRQ
320     106 FORMAT(( RSODD=(,F11.4,8X,(RSHFRQ=(,F10.4,8X,(RSLFRQ=(,F10.4)
            WRITE(6,103)PLPOS,PLNEG
        103 FORMAT(( PLPOS=(,F11.4,8X,(PLNEG=(,F11.4)
            WRITE(6,109)PLNET
        109 FORMAT(( PLNET=(,F11.4)
325         GO TO 110
          1 CONTINUE
            STOP
            END
```

Figure 2. Listing of main program.--Continued

18

(1) Input data for this program are in the form of digital magnetic-tape records of 4,100 values. The first 4 values of the records are the gage number, month, day, and time of the observations; the last 4,096 values are the time-series pressure values of the wave gage. In the present program the wave gage pressures are stored in thousandths of a foot (head) water at 0.25-second intervals. Subroutine BUF reads time-series data into array CNTL, where it is averaged to provide 1,024 time-series values of $\Delta t = 1$ second spacing. Units are also divided by 1,000 to convert values to feet (head) of water.

(2) The date groups of record 1 and record 2 are compared to ensure that times of records are simultaneous; if the times are not, the program searches the record file until this condition is met. The two records are than checked for proper sequence to ensure that gage 1 is analyzed first. Subroutine SWITCH switches arrays if they are not in proper order.

(3) Each of the two 1,024 value time series is then analyzed for average values which are printed out along with the average depth of water at each gage site. The average value of each of the time-series records is again averaged and is added to the height of the gages above the bottom to obtain the water depth:

$$\text{DEPTH} = \frac{\text{AVERAGE 1} + \text{AVERAGE 2}}{2} + B$$

in which AVERAGE 1 is the average of time series $1 = a_1(0)$, AVERAGE 2 the average of time series $2 = a_2(0)$, and B the height of sensors above the bottom.

An option to apply a weighting function $w(j)$ ($= W(I)$ in program) has been incorporated before the FFT subroutine is called. In this particular program a cosine bell weighting function has been incorporated. If the data window option is selected, the two time-series data records, which are read into F1R and F2R arrays, are multiplied by the following weighting function (cosine bell)

$$w(j) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi j}{N}\right)\right]$$

where j is the time step number and N the number of data points in series. If no weighting function is desired in analysis set $w(j) = 1.0$, which is the "box car" weighting function.

As the cosine bell function reduces the total energy content of the waves, the final energy obtained from the FFT must be rescaled up to the proper value. This is accomplished by scaling up the time-series pressure values by the ratio

$$R = \frac{\text{Unwindowed energy}}{\text{Windowed energy}} = \sqrt{\frac{\langle p^2 \rangle}{\langle p'^2 \rangle}}$$

as discussed in equation (4).

19

(4) Cospectra and quad-spectra of the gages are computed using the following relationships (note in computer program index, I is used for frequency counter, n):

Cospectra = C12(I) = F1R(I)*F2R(I) + F1I(I)*F2I(I)

Quad-spectra = Q12(I) = F1R(I)*F2I(I) - F2R(1)*F1I(I)

in which F1R and F1I are the real and imaginary parts of complex transforms of time series 1: F2R and F2I are the real and imaginary parts of complex transforms of time series 2.

(5) Wave angle is calculated in accordance with equation (19).

$$\theta(n) = \theta = \frac{\beta}{\text{arcosine}} \left[\frac{1}{k(n)\ell} \cdot \arctan \frac{Q12(n)}{C12(n)} \right]$$

where $k(n)$ is the wave number calculated via linear wave theory, ℓ the spacing of gages, and β the difference in alinement of gages and shoreline in Figure 1.

Due to energy leakage problems in spectra, impossible wave angles can result [wave angles with $(1/k(n)\ell \arctan Q12(n)/C12(n))$ greater than 1.0]. When this happens, energy is lumped into a separate category for later scaling up of the longshore energy flux.

(6) The high frequency cutoff in this particular program has been set at 2.09 radians per second, which corresponds to a period of 3 seconds or NYFR = 342. This value can be reset in the main program by adjustment of NYFR where

$$\text{NYFR} = \frac{N\Delta t}{T_{HF}}$$

and N is the number of data points in time series, Δt the spacing in time of data points, and T_{HF} the high frequency cutoff period. The spatial aliasing frequency is computed in subroutine HFC.

Energy between the spatial aliasing frequency and the high frequency cutoff is put into a special category and used to scale up the final longshore energy flux.

(7) Each frequency contribution to the onshore energy flux is calculated for the gage site location as follows:

Onshore energy flux = $2\gamma \ |F_\eta(n)|^2 \ C_g(n) \ \cos \ [\theta(n)]$

20

where

$|F_\eta(n)|$ = modulus of the complex amplitude spectra of wave elevation above mean surface at gage site

$C_g(n)$ = group wave speed at gage site

$\Theta(n)$ = Θ = angle of wave direction (see Fig. 1)

γ = specific weight of seawater

The onshore energy flux is then summed to obtain the total onshore energy flux. In the program, onshore energy flux/γ = HG2.

(8) Breaking wave height at the shoreline is determined from the mean square onshore energy flux via a linear theory wave transformation formula which can be simplified to

$$H_b = \left[\sum_{n=1}^{N/2} 16 |F_\eta(n)|^2 \, C_g(n) \, \cos \Theta(n) \right]^{0.4} \left(\frac{GB}{g} \right)^{0.2}$$

where GB is the wave height-to-water depth ratio at breaking and g the acceleration of gravity.

The choice of GB is up to the user although a value of GB = 1.42 has been found by Komar and Gaughan (1972) to best fit wave tank data for breaking wave heights for monochromatic waves. In the present program, GB has been set equal to 0.78 but can be readily changed.

The breaking wave water depth is calculated from the equation

$$\frac{H_b}{d_b} = GBP$$

where d_b is the wave breaking water depth and GBP the ratio of wave height to water depth at breaking.

In this case a different value of the ratio of breaking wave height to water depth can be used in the program for obtaining the proper water depth. An assumed value of GBP = 0.78 from the solitary wave theory in the SPM is used.

Linear wave celerity is assumed and breaking wave celerity is estimated as

$$C_b = \left(g \, \frac{H_b}{GBP} \right)^{0.5}$$

The breaking wave height and celerity calculated in this approach apply to all frequencies.

21

(9) Frequency-by-frequency modification of wave angles is made assuming linear wave theory, Snell's law, and parallel bottom contours offshore. The breaking wave angle, $\Theta_b(n)$, is calculated from

$$\Theta_b(n) = \arcsin \left[\frac{C_b(n) \sin \Theta_r(n)}{C_r} \right]$$

where the subscript r refers to the reference gage location.

(10) Longshore energy flux is calculated for each frequency component (except the special cases discussed in Sec. II) using the equation

$$P_{\ell s}(n) = \gamma |F_n(n)|^2 C_{gb}(n) \sin 2\Theta_b(n)$$

and is summed up to obtain a net longshore energy flux.

(11) The value of the net longshore energy flux is multiplied by a factor R which scales up the total energy in the spectrum (below the high frequency cutoff). The equation for scaling factor R is

$$R = \frac{1}{(1 - RTOT)}$$

where RTOT = RSODD + RSHFRQ when RSODD is the percent of energy in low frequency bands for which impossible values of the cosine function are calculated, and RSHFRQ is the percent of energy between spacial aliasing frequency and high frequency cutoff.

The final result of analysis of the two gage records for the net longshore energy flux PLNET is printed out, as well as specific frequencies for which impossible directional results occur and frequencies at which more than 2.5 percent of the total wave energy is found.

IV. SUBROUTINE DOCUMENTATION

1. <u>FFT Subroutine.</u>

The sampled time function, $f(j)$, will be expressed as

$$f(j) = \sum_{n=0}^{N-1} F(n) \exp(in\omega_1 j \Delta t)$$

in which

$$\omega_1 = \frac{2\pi}{\text{record length}} = \frac{2\pi}{T} = \frac{2\pi}{N\Delta t}$$

$$t_j = j\Delta t = \text{a discrete time where } j \text{ is the integer time step}$$

$$F(n) = a(n) - ib(n)$$

$$a\left(\frac{N}{2+n}\right) = a\left(\frac{N}{2-n}\right), \ n \neq 0, \ \frac{N}{2}$$

$$b\left(\frac{N}{2+n}\right) = -b\left(\frac{N}{2-n}\right), \ n \neq 0, \ \frac{N}{2}$$

$$a(0) = \text{mean of sampled record}$$

$$b(0) = b\left(\frac{N}{2}\right) = 0$$

Because negative indexes are not readily handled by most FORTRAN compilers, the summation extends over the interval $0 \leq n \leq N - 1$, rather than over the symmetric interval $-N/2 \leq n \leq N/2$. From the definition of the coefficients above, it is clear that the coefficients $a(n)$ and $b(n)$ for $n > N/2$ contain no additional information.

The inverse relationship completing the FFT pair is

$$F(n) = \frac{1}{N} \sum_{j=1}^{N} f(j) \exp(-in\omega_1 j\Delta t)$$

As an enumeration of the complex FFT coefficients, suppose the series of 8 values is considered, $N = 8$. The coefficients would be

$$F(0) = a(0)$$

$$F(1) = a(1) - ib(1), \ F(7) = a(7) - ib(7) = a(1) + ib(1)$$

$$F(2) = a(2) - ib(2), \ F(6) = a(6) - ib(6) = a(2) + ib(2)$$

$$F(3) = a(3) - ib(3), \ F(5) = a(5) - ib(5) = a(3) + ib(3)$$

$$F(4) = a(4)$$

This pattern prevails for all sets of FFT coefficients, regardless of the value of N. Both $F(0)$ and $F(N/2)$ are real and, as noted previously, the coefficients $F(n)$ for $n > N/2$ really contain no additional information. The FFT subroutine used here requires that the number of data points, N, provided be an integral power of 2, i.e.,

$$N = 2^K$$

where K is an integer. Thus analyses of 512, 1,024, or 2,048 data points (K = 9, 10, 11) would be suitable with this subroutine.

23

The following two requirements are satisfied in the FFT subroutine.

(a) By operating on the sampled function, obtaining the F(n) coefficients and carrying out the inverse FFT (FFT^{-1}), the original time function is recovered. Schematically,

$$f(j) \rightarrow \boxed{\text{FFT}} \rightarrow F(n) \rightarrow \boxed{\text{FFT}^{-1}} \rightarrow f(j)$$

(b) The mean square of the sampled time function is equal to the sum of the squares of the moduli of the FFT coefficients, F(n), i.e.,

$$\frac{1}{N} \sum_{j=1}^{N} [f(j)]^2 = \sum_{n=0}^{N-1} |F(n)|^2$$

a. <u>Calling Statement</u>: SUBROUTINE FFT (FR, FI, K, ICO) (see Fig. 3). FR, FI = real and imaginary coefficients in

$$F(n) = FR(n) - iFI(n)$$

$$K = \text{power of two (i.e., } N = 2^K)$$

$$ICO = \text{control whether FFT or } (\text{FFT})^{-1}$$

operation is desired if

$$ICO \begin{cases} = 0 \rightarrow \text{FFT} \\ = 1 \rightarrow (\text{FFT})^{-1} \end{cases}$$

When entering the subroutine, FR is the time sequence f(j) and FI is arbitrary. When exiting the subroutine, FR and FI are the real and imaginary parts of the complex transform, respectively; e.g., input is

$$K = 5$$

$$ICO = 0$$

$$f(j) = 1.0 + 2.0 \cos \frac{2\pi(j\Delta t)}{32} + 3.0 \cos \frac{4\pi(j\Delta t)}{32}$$

$$- 0.6 \sin \frac{2\pi(j\Delta t)}{32} - 1.4 \sin \frac{4\pi(j\Delta t)}{32}$$

24

```
1         C
          C        FAST FOURIER TRANSFORM SUBROUTINE
                   SUBROUTINE FFT(FR,FI,K,ICO)
                   DIMENSION FR(1),FI(1)
5                  N=2**K
                   IF(ICO.EQ.0) GO TO 10
                   DO 8 I=1,N
          8 FI(I)=-FI(I)
          10 CONTINUE
10                 MM=0
                   NNCN=1
                   DO 2 M=1,NN
                   L=N
          1 L=L/2
15                 IF(MM+L.GT.NN) GO TO 1
                   MM=MOD(MM,L)+L
                   IF(MM.LE.M) GO TO 2
                   TR=FR(M+1)
                   FR(M+1)=FR(MM+1)
20                 FR(MM+1)=TR
                   TI=FI(M+1)
                   FI(M+1)=FI(MM+1)
                   FI(MM+1)=TI
          2 CONTINUE
25                 L=1
          3 IF(L.GE.N) GO TO 7
                   ISTEP=2*L
                   EL=L
                   DO 4 M=1,L
30                 A=3.1415926535*FLOAT(1-M)/EL
                   WR=COS(A)
                   WI=SIN(A)
                   DO 4 I=M,N,ISTEP
                   J=I+L
35                 IF(ICO.EQ.1) GO TO 11
                   TR=WR*FR(J)-WI*FI(J)
                   TI=WR*FI(J)+WI*FR(J)
                   GO TO 12
          11 TR=WR*FR(J)+WI*FI(J)
40                 TI=WR*FI(J)-WI*FR(J)
          12 FR(J)=FR(I)-TR
                   FI(J)=FI(I)-TI
                   FR(I)=FR(I)+TR
          4 FI(I)=FI(I)+TI
45                 L=ISTEP
                   GO TO 3
          7 CONTINUE
                   AN=N
                   IF(ICO.EQ.1) GO TO 6
50                 DO 5 I=1,N
                   FR(I)=FR(I)/AN
          5 FI(I)=-FI(I)/AN
          6 RETURN
                   END
```

Figure 3. Listing of FFT subroutine.

b. **Data Input to Program.**

f(j) values at	6.000	5.080	3.750	2.184
$\Delta t = 1$ second	0.590	−0.829	−1.900	−2.506
intervals	−2.600	−2.215	−1.451	−0.465
(32 values)	0.562	1.445	2.034	2.229
	2.000	1.391	0.513	−0.475
	−1.390	−2.054	−2.322	−2.109
	−1.400	−0.257	1.188	2.755
	4.238	5.438	6.189	6.386
FR =	6.000	5.080	3.750	2.184
(32 values)	0.590	−0.829	−1.900	−2.506
	−2.600	−2.215	−1.451	−0.465

		0.562	1.445	2.034	2.229
		2.000	1.391	0.513	−0.475
		−1.390	−2.054	−2.322	−2.109
		−1.400	−0.257	1.188	2.755
		4.238	5.438	6.189	6.386
FI =		0.000	0.000	0.000	0.000
(32 values)		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000

c. <u>Calling Statement</u>: FFT (XR, XI, 5, 0).

Output:		1.000	1.000	1.500	0.000
a(n) coefficients		0.000	0.000	0.000	−0.000
(32 values)		−0.000	−0.000	−0.000	−0.000
		−0.000	−0.000	−0.000	−0.000
		−0.000	−0.000	−0.000	−0.000
		−0.000	−0.000	−0.000	−0.000
		−0.000	0.000	0.000	0.000
		0.000	0.000	1.500	1.000
b(n) coefficients		0.000	−0.300	−0.700	−0.000
(32 values)		−0.000	−0.000	−0.000	−0.000
		−0.000	−0.000	−0.000	−0.000
		−0.000	−0.000	−0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.000	0.000
		0.000	0.000	0.700	0.300

Δt (time step) = 1 second in above example.

2. HFC Subroutine.

This subroutine resets the spatial aliasing frequency cutoff to a higher frequency than would be the case for normal incidence of waves to gage pair. In the present version of this subroutine, it has been assumed that the maximum angle which the wave crests can make with the gage pair axis is 45°. The spatial aliasing criteria are expressed in Figure 1, where for proper resolution of wave direction the following criteria must be met

$$\ell\cos\,[\theta(n) - \beta] < \frac{L}{2}$$

$$k(n)\,\ell\cos\,[\theta(n) - \beta] < k(n)\,\frac{L}{2}$$

26

The proper spatial aliasing frequency to correspond with the spacial aliasing wave number cutoff is found from the normal wave dispersion relationship.

Calling Statement: HFC (DEPTH, S, DELTT, N, NSALFR) (see Fig. 4).

DEPTH = depth of water at gage site (from main program)

S = spacing of wave gage pair (from main program) (= ℓ in text)

DELTT = time-step increment between values in time series analyzed (from main program)

N = exponent of 2 describing number of time series values (from main program)

NSALFR = integer number for spatial aliasing frequency cutoff

```
1        C
         C    SUBROUTINE HFC(HIGH FREQUENCY CUTOFF/SPACIAL ALIASING FREQUENCY)
         C       RESETS ALIASING CUTOFF TO HIGHER FREQUENCY
         C       BASED ON ASSUMED MAXIMUM WAVE ANGLE
5        C    SUBROUTINE HFC(DEPTH,S,DELTT,N,NSALFR)
         C    SPACIAL ALIASING ASSUMES WAVE ANGLES LESS THAN 45 DEGREES
              XKS=3.14159/0.707
              XKH=XKS*DEPTH/S
              SIGSU=32.2*(XKH/DEPTH)*TANH(XKH)
10            SIGHF=SQRT(SIGSU)
              RECLN=FLOAT(N)*DELTT
              NSALFR=SIGHF*RECLN/6.283
              RETURN
              END
```

Figure 4. Listing of HFC subroutine.

3. SWITCH Subroutine.

This subroutine is set up to interchange time-series data arrays in the instance when gage 2 data are processed before gage 1 data (see Fig. 5). If the first gage record processed is not equal to the appropriate number of the gage, as specified in main program, data arrays of first and second gage records are interchanged.

```
1        C
         C    SUBROUTINE SWITCH EXCHANGES LOCATIONS OF TIME SERIES DATA TO ASSURE
         C       GAGE1 IS STORED IN FIRST ARRAY AND GAGE2 IN SECOND
              SUBROUTINE SWITCH(M1,M2,F1R,F2R)
5             DIMENSION F1R(1024),F2R(1024),F3R(1024)
              M3=M1
              M1=M2
              M2=M3
              DO 10 I=1,1024
10            F3R(I)=F1R(I)
              F1R(I)=F2R(I)
              F2R(I)=F3R(I)
         10 CONTINUE
              RETURN
15            END
```

Figure 5. Listing of SWITCH subroutine.

4. WVLEN Subroutine.

This subroutine accepts wave period and water depth as input and calculates wave number as output via a Newton-Raphson iteration.

27

Calling Statement: WVLEN (DPT, PER, XKH) (see Fig. 6).

DPT = water depth (from main program)

PER = wave period (from main program)

XKH = wave number * water depth (calculated in subroutine)

```
 1          C
            C      WAVE LENGTH ITERATION SUBROUTINE--THIS SUBROUTINE CALCULATES WAVELENGTH
            C        VIA NEWTON-RAPHSON ITERATION USING PERIOD,WATER DEPTH INPUT
            C      PER=WAVE PERIOD
 5          C      DPT=WATER DEPTH
            C      XKH=WAVE NUMBER*WATER DEPTH
                   SUBROUTINE WVLEN(DPT,PER,XKH)
                   XKHO=(6.2831853/PER)**2*DPT/32.2
                   IF(XKHO-6.3)2,1,1
10               1 XKH=XKHO
                   GO TO 9
                 2 XKH=SQRT(XKHO)
                 3 SH=SINH(XKH)
                   CH=COSH(XKH)
15                 EPS=XKHO-XKH*SH/CH
                   SLOPE=-XKH/CH**2-SH/CH
                   DXKH=EPS/SLOPE
                   IF(ABS(DXKH/XKH)-0.0001)9,9,4
                 4 XKH=XKH+DXKH
20                 GO TO 3
                 9 CONTINUE
                   RETURN
                   END
```

Figure 6. Listing of WVLEN subroutine.

5. BUF Subroutine.

This subroutine is set up to read in wave gage files from magnetic tape. The data records consist of arrays of 4,100 values, the first four of which are the gage number, month, day, and time of wave record. The remaining 4,096 values represent pressures in thousandths of a foot (head) water. The data are returned to main program as a wave gage number-date series and a time series of 4,096 values of pressure in feet (head) of water. Two records are processed in one pass.

Calling Statement: BUF (MGAGE, MONTH, MDAY, MTIME, CNTL, IDATE, END) (see Fig. 7).

MGAGE = number of gage (read from tape)

MONTH = month of observation (read from tape)

MDAY = day

MTIME = time

CNTL = control array of 4,096 pressure values in feet (head) of water returned to main program

IDATE = summed time group for time comparison between gages

END = logical end

28

```
 1        C
          C     SUBROUTINE BUF READS IN WAVE GAGE DATE INFO AND TIME SERIES DYNAMIC
          C        PRESSURE VALUES IN FEET HEAD OF WATER
          C     THIS SUBROUTINE READS 4096 TIME SERIES VALUES AND AVERAGES TO OBTAIN
 5        C        1024 VALUES FOR MAIN PROGRAM ANALYSIS
          C     MGAGE=GAGE NUMBER
          C     MONTH=MONTH OF RECORDING
          C     MDAY=DAY OF RECORDING
          C     MTIME=TIME OF RECORDING
10        C     REAL=ARRAY OF AVERAGED TIME SERIES VALUES
                SUBROUTINE BUF(MGAGE,MONTH,MDAY,MTIME,REAL,IDATE,END)
                DIMENSION CNTL(4096),IA(5000)
                DIMENSION REAL(1024)
                LOGICAL END
15              DO 12 J=1,4096
                CNTL(J)=0.0
             12 CONTINUE
                BUFFER IN(9,1)(IA(1),IA(5000))
                IF(UNIT(9))10,20,30
20           20 PRINT 11,(IA(I),I=1,4)
             11 FORMAT(( PARITY ERROR ON(,4I7)
             10 CONTINUE
                MGAGE=IA(1)
                MONTH=IA(2)
25              MDAY=IA(3)
                MTIME=IA(4)
                IDATE=IA(2)+IA(3)+IA(4)
                K=4
                DO 25 J=1,4096
30              K=K+1
                CNTL(J)=IA(K)
             25 CNTL(J)=CNTL(J)/1000.
                DO 26 J=4088,4096
             26 CNTL(J)=CNTL(4087)
35              J=1
                DO 27 L=1,1024
                REAL(L)=(CNTL(J)+CNTL(J+1)+CNTL(J+2)+CNTL(J+3))/4.
                J=J+4
             27 CONTINUE
40              RETURN
             30 END=.TRUE.
                RETURN
                END
```

Figure 7. Listing of BUF subroutine.

V. SAMPLE OUTPUT

Three examples of output are presented for different dates for the wave gage pair at Channel Islands Harbor (Fig. 8). The year the data was taken was 1975.

The first set of frequencies lists amplitude modules squared of wave data having impossible direction results. The sum total of this energy (in decimal percent) is listed as the quantity RSODD in the variable output at the bottom of the output. In the case of the wave data taken on 7-26-1600, the incoherent data amounted to 0.004 (0.4 percent) of the total energy in the wave record.

The second set of frequencies listed provides the wave direction for the frequency bands having a significant part of the energy (≥ 2.5 percent). In the case of the wave record taken on 7-26-1600, it is seen that the wave angle is reasonably consistent from the frequency-to-frequency band and is approximately 0.70 radian (40.1°).

The variable list provided at the bottom of the sampled output gives values of most importance in the analysis of wave information for longshore energy flux. The longshore energy flux output is in pounds per second units; the output in the first example is 89.23 pounds per second.

29

Example 1

GAUGE NO.	MONTH	DAY	TIME
311	7	26	1600
312	7	26	1600

I	SIGMA(I)	FMDSQ(I)
3	.018408	.000025
4	.024544	.000014
5	.030680	.000027
7	.042951	.000036
9	.055223	.000012
10	.061359	.000021
11	.067495	.000048
14	.085903	.000005
24	.147262	.000127
25	.153398	.000041
27	.165670	.000002
29	.177942	.000099
30	.184078	.000040
32	.196350	.000066
33	.202485	.000029
42	.257709	.000014
45	.276117	.000041
65	.398835	.000093

I	SIGMA(I)	PCT	THETA(I)
67	.41110685	.03904604	.70276903
68	.41724277	.04032988	.77995085
73	.44792239	.02831251	.88437628
74	.45405831	.10395194	.69738613
75	.46019424	.06890760	.69198090
78	.47860201	.02500015	.71912354
79	.48473793	.04457282	.63791396

NBALFR=			201
DEPTH OF WATER AT GAUGE SITE=			23.2
AVG1=	21.411	AVG2=	19.999
SUM1=	.229	SUM2=	.234
NSUM1=	.084	NSUM2=	.086
RATIO1=	2.730	RATIO2=	2.729
SUMEN=	.18433938		
BREAKING WAVE HEIGHT HB=			3.03
BREAKING WAVE CELERITY CB=			11.18
RBODD=	.0040	RBMFHG=	.2413	RBLFRU=	.0170
PLPOS=	94.6421	PLNEG=	=5.4150
PLNET=	89.2271	

Figure 8. Three examples of output for wave gage pair at Channel Islands Harbor.

Example 2

GAUGE NO.	MONTH	DAY	TIME
311	7	26	1800
312	7	26	1800

I	SIGMA(I)	FMDSO(I)
1	.006136	.000169
2	.012272	.000099
3	.018408	.000007
5	.030680	.000013
8	.049087	.000066
9	.055223	.000164
10	.061359	.000038
11	.067495	.000000
13	.079767	.000168
14	.085903	.000201
15	.092039	.000137
16	.098175	.000114
18	.110447	.000055
19	.116583	.000061
23	.141126	.000004
26	.159534	.000072
28	.171806	.000006
30	.184078	.000020
31	.190214	.000006
42	.257709	.000028
58	.355884	.000064

I	SIGMA(I)	PCT	THETA(I)
68	.41724277	.02575229	.75283813
75	.46019484	.04845316	.65863227
76	.46633016	.03141753	.69562037
77	.47246608	.00237300	.73494726

```
NSALFR=                                    203
DEPTH OF WATER AT GAUGE SITE=              24.0
AVG1=      22.246          AVG2=          20.808
SUM1=        .299          SUM2=            .293
WSUM1=       .084          WSUM2=           .078
RATIO1=    3.579           RATIO2=        3.741
SUMEN=      .31019470
BREAKING WAVE HEIGHT HB=                   3.61
BREAKING WAVE CELERITY CB=               12.22
RSODD=       .0048         RSHFRQ=         .3742    RSLFRU= .0114
PLPOS=    125.7135         PLNEG=       -25.6734
PLNET=    100.0401
```

Figure 8.   Three examples of output for wave gage pair at Channel
Islands Harbor.--Continued

31

Example 3

GAUGE NO.	MONTH	DAY	TIME
311	7	26	2000
312	7	26	2000

I	SIGMA(I)	PMDSQ(I)
1	.006136	.000930
2	.012272	.000397
3	.018408	.000179
4	.024544	.000052
7	.042951	.000013
8	.049087	.000021
10	.061359	.000014
12	.073631	.000074
13	.079767	.000085
17	.104311	.000080
19	.116583	.000008
23	.141126	.000011
32	.196350	.000009
34	.208621	.000100
35	.214757	.000104
36	.220893	.000019
42	.257709	.000044
55	.337476	.000136
71	.435651	.001058

I	SIGMA(I)	PCT	THETA(I)

```
NBALFR=                          202
DEPTH OF WATER AT GAUGE SITE=    23.5
AVG1=      21.702      AVG2=     20.287
SUM1=        .253      SUM2=       .266
WSUM1=       .073      WSUM2=      .081
RATIO1=     3.460      RATIO2=    3.299
SUMEN=       .35101917
BREAKING WAVE HEIGHT HB=         3.66
BREAKING WAVE CELERITY CB=      12.29
RSODD=       .0095      RSHFRU=     .5675      RSLFRU= .0142
PLPOS=     135.5367     PLNEG=   -40.2297
PLNET=      95.3070
```

Figure 8. Three examples of output for wave gage pair at Channel
          Islands Harbor.--Continued

LITERATURE CITED

BRUNO, R.O., et al., "Longshore Sand Transport Study at Channel Islands Harbor, California," TP 81-2, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Mar. 1981.

HARRIS, D.L., "Finite Spectrum Analyses of Wave Records," *Proceedings of the International Symposium on Ocean Wave Measurement and Analysis*, American Society of Civil Engineers, 1974, pp. 107-124 (also Reprint 6-74, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., NTIS A002 114).

KOMAR, P.D., and GAUGHAN, M.K., "Airy Wave Theory and Breaker Height Prediction," *Proceedings of the 13th Coastal Engineering Conference*, American Society of Civil Engineers, Vol. 1, 1972, pp. 405-418.

THOMPSON, E.F., "Energy Spectra in Shallow U.S. Coastal Waters," TP 80-2, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Feb. 1980.

U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3d ed., Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.

Walton, Todd L.
   Computer algorithm to calculate longshore energy flux and wave
direction from a two pressure sensor array / by Todd L. Walton and
Robert G. Dean.--Fort Belvoir, Va. : U.S. Army, Corps of Engineers,
Coastal Engineering Research Center ; Springfield, Va. : available
from NTIS, 1982.
   [31] p. : ill. ; 28 cm.--(Technical paper / U.S. Coastal Engineer-
ing Research Center ; no. 82-2). Cover title. "August 1982."
   A FORTRAN IV computer program (written for the CERC Longshore Sand
Transport Research Program and designed to accept data in the CERC
magnetic-tape format of record lengths consisting of 4,100 values)
is used to analyze wave data collected at Channel Islands Harbor,
California. Steps in an analysis of wave data and sample outputs for
some wave records from a wave gage pressure sensor pair are given.
   1. Computer programs. 2. Wave direction measurement. 3. Wave
gages. 4. Sediment transport. 5. Wave spectra. 6. Longshore energy
flux. I. Title. II. Dean, Robert G. III. Series: Technical paper
(Coastal Engineering Research Center (U.S.)); no. 82-2.
TC203
  .U581tp   no. 82-2   627